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Notch brittleness of ductile glassy polymers under plane strain

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The brittle fracture behaviour of ductile glassy polycarbonate, poly(vinyl chloride) and poly(methyl methacrylate) notched bars subjected to plane strain tension has been studied at varying strain rates. Morphological observations of thin sections and fracture surfaces revealed that a disc-type craze was nucleated at the tip of the plastic zone which spread from the notch root. A slip-line theory modified so that the yield criterion is influenced by a hydrostatic stress component allows the calculation of the stress components at the elastic–plastic boundary, where the hydrostatic stress is highest, from the knowledge of the location of the fracture origin. An analysis of the data resulted in the conclusion that Orowan's analysis for notch brittleness is appropriate.

1. Introduction

The question of a mechanical criterion for fracture in glassy polymers has been extensively investigated in recent years. Several criteria based on linear fracture mechanics have been suggested for the fracture behaviour of either brittle or quasibrittle polymers when yielding is restricted to a small region around the crack tip. In many practical cases, however, the majority of polymers are ductile and exhibit considerably non-linear behaviour. In these polymers, the plastic deformation at the vicinity of the crack tip exceeds even the limit of validity of the theory of non-linear fracture mechanics. When, for a polymer in the form of thick specimens, the yielding occurs at the crack tip, the elastic material outside the yielding zone provides a constraint to the zone and the triaxial state of stress, which strongly affects fracture behaviour, is introduced by this constraint at the elastic–plastic boundary in the material.

There has been less published on the fracture mechanisms of such a fracture process. Grade and Weiss [1] made a study of fracture nucleation in slow tensile tests of double edge-notched polycarbonate bars and calculated a critical normal stress for crack nucleation at the elastic–plastic boundary by using Hill's [2] slip line field analysis of plane strain yielding at a circular notch in a rigid–perfectly plastic material. In a recent study of impact fracture of notched polycarbonate bars Mills [3] has shown that internal crazes and cracks are nucleated at the elastic–plastic boundary and calculated the triaxial stress components for craze nucleation. Independently, Ishikawa *et al.* [4] made a similar study of an internal craze and brittle fracture of notched polycarbonate bars subjected to slow three-point bending. They have also used the Hill's slip line field thoery and concluded that a dilatational stress produced by the stress triaxiality at the elastic–plastic boundary is responsible for craze and fracture initiation. There is theoretical backing for their conclusion since recognition that crazing requires void formation leads to the suggestion that a craze criterion must involve a dilatational stress component.

The present investigation was undertaken to extend the three-point bending results to crazing and fracture behaviour of notched specimens of polycarbonate, poly(vinyl chloride) and poly(methyl methacrylate) subjected to plane strain tension and to provide more experimental evidence of the dilatational fracture mechanism in ductile glassy polymers.

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2. Experimental details

2.1. Material and specimens

The materials used were commercial grades of extruded polycarbonate (PC, Panlite), calendered poly(vinyl chloride) (PVC, Hishiplate), and cast poly(methyl methacrylate) (PMMA, Acrylite) sheet of 10 mm thickness. Rectangular strips of 10 mm x 10 mm cross-section and 160 mm long were cut from the sheets. They were metallographically polished by hand to prevent formation of surface crazes, using wet billiard cloth with a polishing solution of Cr_2O_3. Parallel-sided double notches of 0.5 mm radius and 3.0 mm long were introduced by making a saw cut in the centre of both edges. In order to avoid the development of a layer of oriented polymer on the surface of the saw cuts caused by an increase in temperature, specimens were cooled with water during cutting. The notch root of each specimen was also polished using metallographic polishing techniques. PC and PMMA specimens were annealed for 24 h at the glass transition temperatures, 425 and 403° K, respectively, to eliminate residual strain. After annealing, they were either cooled slowly from the annealing temperature to room temperature at a rate of $10\,K\,h^{-1}$ or were quenched into ice water. PVC specimens were not annealed and used in the as-received form since delamination occurred during heat-treatment. Therefore, some residual orientation is unfortunately left in PVC specimens.

2.2. Experimental procedure

Tensile tests were carried out with an Instron-type testing machine (Auto Graph, Shimazu DSS-5000) at varying cross-head speeds. The gauge length of the specimen was 100 mm long, but since the effective gauge length for notched specimens is of the order of the root radius the strain rate at the notch root can be two orders of magnitude higher than that which is measured in a smooth test specimen, for the same speed of cross-head elongation [5]. The critical shear stress for yielding was obtained by carrying out the simple uniaxial compression tests on the same specimens at varying cross-head speeds. Rectangular specimens of 10 mm x 10 mm and 50 mm long were compressed using the Auto Graph. All tests were made in air at 293 K and 65% relative humidity.

Microscopic observations were made on the specimen unloaded during deformation and after fracture to examine the plastic deformation region and to determine the position of fracture origin.

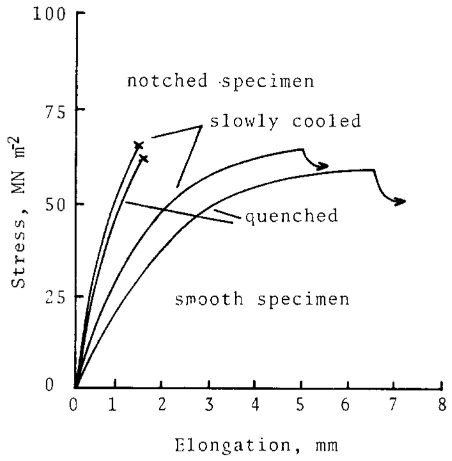

Figure 1 Comparison of stress–elongation curves of notched PC with those of smooth PC.

3. Results and discussion

3.1. Observations of the plastic deformation and fracture processes

Fig. 1 shows typical stress–elongation curves for smooth and double edge-notched specimens of slowly cooled and quenched PC at a cross-head speed of $0.5\,mm\,min^{-1}$. The stress for a notched specimen is the net stress which is calculated from the cross section between the notches. For both smooth specimens fracture occurred after the development of a neck over the entire gauge length. The presence of elastic constraint due to triaxial stress at the root of the notch and higher effective strain rate for the notched specimens apparently make them stiffer than the smooth specimens. In notched specimens deformation initiated around the notch root at a stress level of about 0.4 to 0.5 of the fracture stress. As the applied stress was increased, the plastic deformation spread from the notch root; and then a shallow

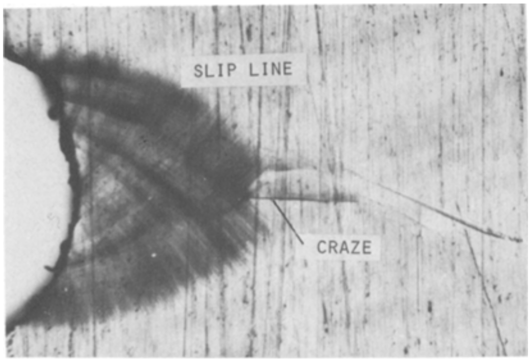

Figure 2 Normal view of shear band and fracture origin (craze) ahead of the plastic zone of slowly cooled PC.

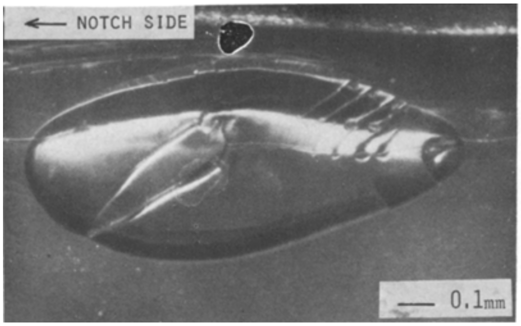

Figure 3 Oblique view of fracture origin ahead of the plastic zone of slowly cooled PC.

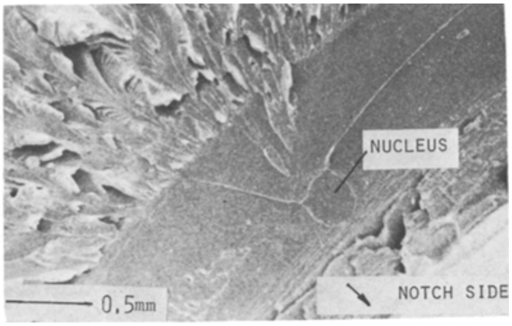

Figure 5 Scanning electron micrograph of fracture surface of slowly cooled PC.

basin was formed at the specimen surface ahead of the notch root. The microscopic observation of a thin longitudinal slice at midsection showed that the plastic deformation zone was composed of many shear bands. When the applied stress reached a value of about 0.9 of the fracture stress, a crack, approximately semi-circular in form in an oblique view, nucleated near the tip of the plastic zone as shown in Figs. 2 and 3. The crack is observed as a series of concentric interference fringes. Fig. 4 shows that the fringe pattern spreads out symmetrically, but, as the size of the crack increases, it becomes asymmetrical and wedge-shaped with an irregular edge next to the elastic–plastic boundary.

Mills [3] suggested that similar disc-shaped cracks nucleated at the elastic–plastic boundary of the Charpy impact tested PC are definitely crazes, and not cracks assumed by Grade and Weiss [1]. His observations were that collimated light could pass through the disc region without total internal reflection when the angle of incidence to the specimen surface was only 60°, and that only one set of the two fringe patterns of the craze and crack was found when the disc was examined by normally incident light. Examination of a refractive index gives a quantitative experimental confidence to Mill's suggestion. The refractive index obtained from the critical angle for total reflection of the disc region of slowly cooled PC was 1.23 as compared to 1.58 for that of the bulk specimen. Using the Lorenz–Lorentz equation, the void content of the disc region is calculated to be about 50%. Kambour [6] has reported that the void content of a craze of PC with ethanol as a crazing agent is 45%.

3.2. Examination of fracture surfaces

The process involved in the fracture mentioned above is reflected in the general appearance of the fracture surfaces. As a typical example, a scanning electron micrograph of the fracture surface of slowly cooled PC is shown in Fig. 5. A smooth region around the nucleus corresponds to the shape the disc at the tip of the plastic zone. It can be noted that there is shear lip along the notch showing that a rapid crack propagated through the shear bands in a zig-zag mode. Outside the smooth region in the width direction of the

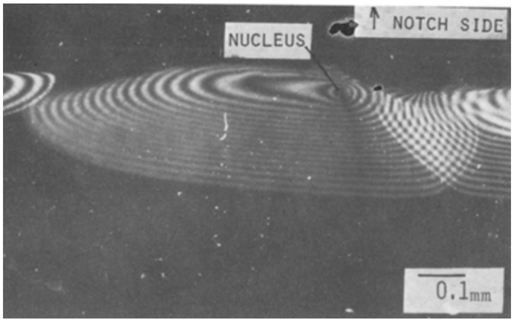

Figure 4 Interference micrograph of a thin section of slowly cooled PC. The micrograph was taken so that the thin section was viewed in reflected Na-D light.

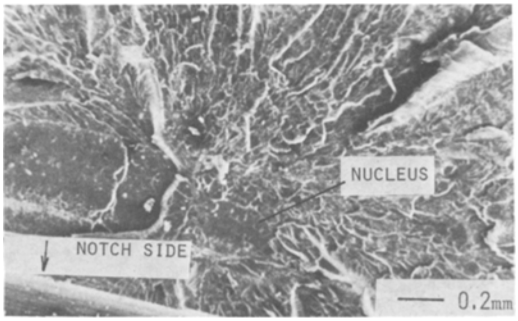

Figure 6 Scanning electron micrograph of fracture surface of PVC.

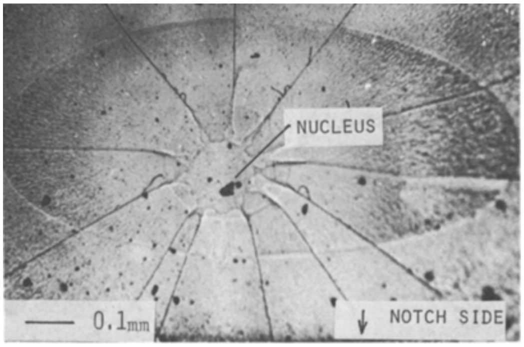

Figure 7 Optical micrograph of fracture surface of quenched PMMA.

specimen the crack is supposed to become unstable and propagate rapidly since a feature of rough surface marks frequently occur in rapid crack propagation in PC [7]. Fig. 6 is a scanning electron micrograph of the fracture surface of PVC. Similar features of the fracture surface are observed although the smooth region which involves a fracture nucleus is less clear than that of PC. Fig. 7 shows an optical micrograph of quenched PMMA. A fracture origin can be easily discerned.

3.3. Critical hydrostatic stress for fracture

As the fracture origin lies at the tip of the plastic zone, the maximum extent of the plastic zone can be measured from the position of the fracture nuclei on the fracture surface. Therefore, a critical hydrostatic stress for fracture can be calculated from the maximum extent of the plastic zone applying Hill's slip-line field theory [2]. When a rigid–perfectly plastic material is assumed, the slip line field of the plastic zone with a deep circular notch in plane strain tension shows logarithmic spirals. The maximum stress, σ_y, along the tensile direction, within the logarithmic spiral region at a distance x ahead of the notch root is given by

$$\sigma_y = 2k[1 + \ln(1 + x/\rho)], \tag{1}$$

where ρ is the notch radius and k is the shear stress for yield in pure shear. If Poisson's ratio is 0.5, the hydrostatic stress s is given by

$$s = \sigma_z = (\sigma_x + \sigma_y)/2 = k[1 + 2\ln(1 + x/\rho)] \tag{2}$$

σ_x and σ_z are the stress components in the notch and thickness directions, respectively.

On the other hand, glassy polymers are known to obey a pressure-dependent yield criterion [8, 9]. For many polymers the yield stress increases when the material is under a hydrostatic pressure. By analogy with the Coulomb yield criterion, the shear stress for yield is given by

$$k = k_0 - \mu s = k_0 - (\cos 2\psi)s, \tag{3}$$

where k_0 and $\mu = \cos 2\psi$ are constants. The μ values of 0.08 for polycarbonate [10], 0.15 for poly(methyl methacrylate) [9], and 0.11 for poly(vinyl chloride) [9] are reported.

The characteristic lines of the double logarithmic

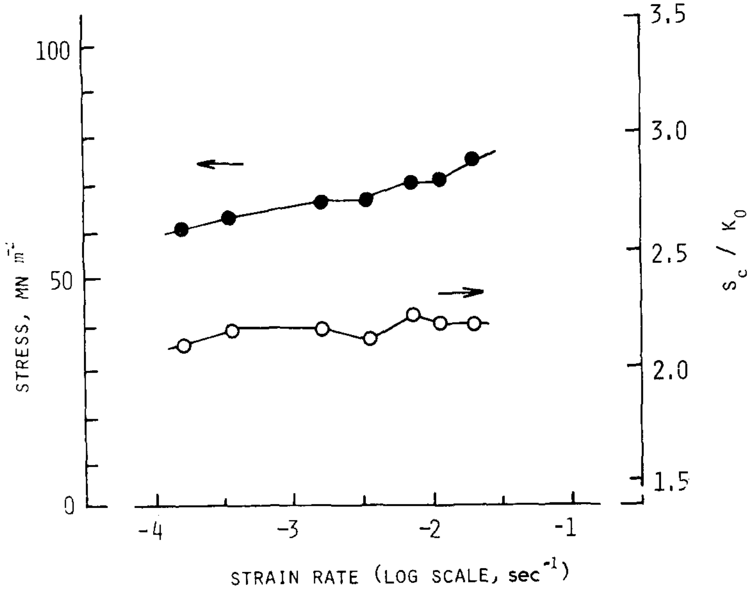

Figure 8 Variation of the critical hydrostatic stress s_c and the ratio of s_c to the shear yield stress k_0 with the logarithm of strain rate for slowly cooled PC.

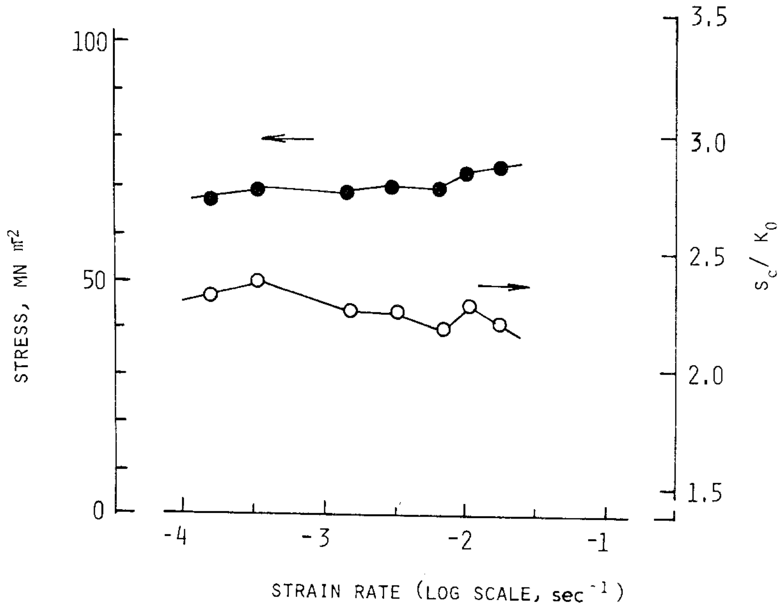

Figure 9 Variation of s_c and s_c/k_0 with the logarithm of strain rate for quenched PC.

spirals for a pressure-dependent yielding material with a deep round notch of radius ρ have the equation in polar co-ordinates r, θ [11]:

$$\theta \cot \psi \pm \ln (r/\rho) = \text{constant}, \tag{4}$$

where the upper and lower signs correspond to α- and β-lines, respectively. The characteristic α and β lines are not the two orthogonal families of curves whose directions at every point coincide with those of the maximum shear stress, but they intersect each other at 2ψ angles. In the specific case of $\cot \psi = 1$, Equation 4 is $\theta \pm \ln (r/\rho) =$ constant, which is a usual relation of the slip lines for a rigid–perfectly plastic material. The hydrostatic stress component in the plastic region for a pressure-dependent yield material is given by

$$s = \frac{k_0}{\mu}\left[1 - \left(1 - \frac{\mu}{1+\mu}\right)\left(\frac{r}{\rho}\right)^{(-2\mu/1+\mu)}\right]. \tag{5}$$

Using Equation 5 the ratio of a critical hydrostatic stress s_c to k_0 can be calculated from the maximum extent of the plastic zone. The constant k_0 can be determined from the yield stress in the uniaxial compression tests were the principal stress components other than σ_u in the compressive direction

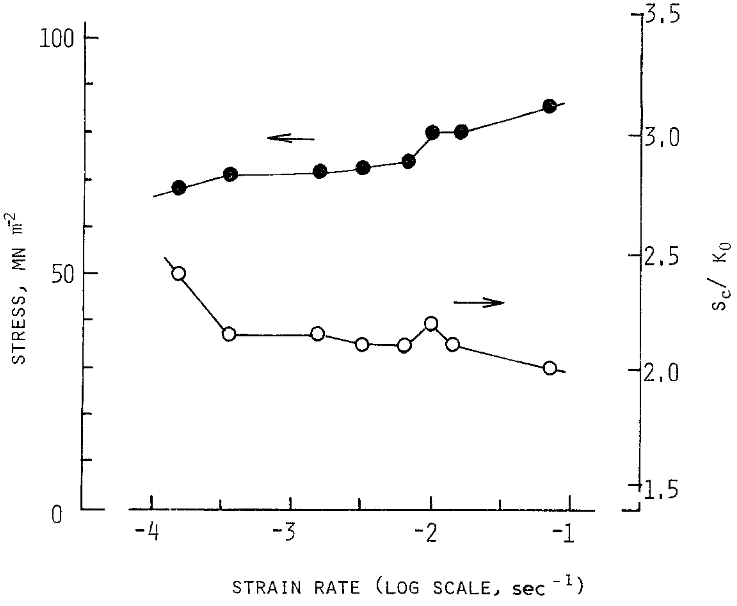

Figure 10 Variation of s_c and s_c/k_0 with the logarithm of strain rate for PVC.

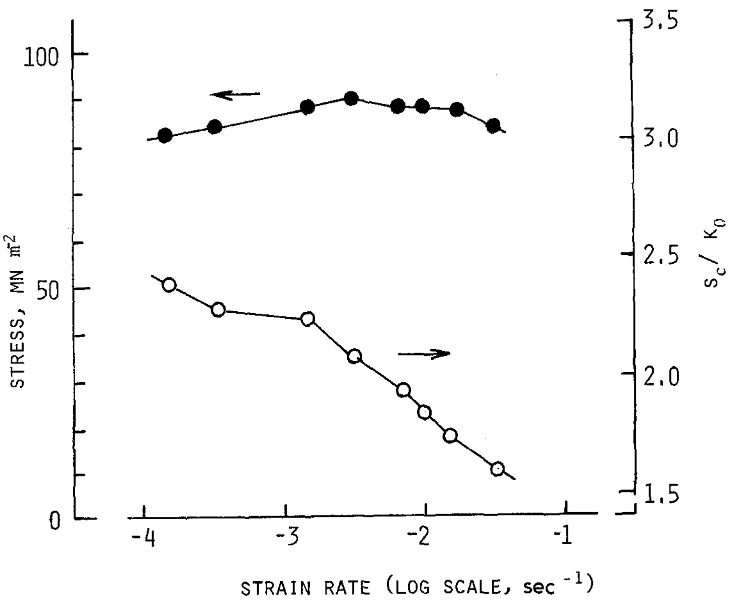

Figure 11 Variation of s_c and s_c/k_0 with the logarithm of strain rate for quenched PMMA.

are zero. On a modified von Mises criterion which is the most appropriate criterion for PMMA, PVC and PC [9, 10], yield will occur when

$$\sigma_u = 3k_0/(1 + 3\mu). \qquad (6)$$

This has been done using the known values of μ mentioned above. Figs. 8 and 9 show the variation of s_c/k_0 and s_c with the logarithm of the strain rate at the notch tip for slowly cooled PC and quenched PC, respectively. Here, the strain rate is the effective strain rate at the notch root which is calculated by dividing the cross-head speed by the notch radius 0.5 mm instead of the nominal gauge length 100 mm. The ratio s_c/k_0 is nearly constant or decreases slightly with increasing strain rate, but the critical hydrostatic stresses for both materials increase with increasing strain rate. Figs. 10 and 11 also show the variation of s_c/k_0 and s_c with the logarithm of strain rate for PVC and quenched PMMA. In slowly cooled PMMA fracture occurred at the surface of the notch root before plastic deformation initiated. The ratio s_c/k_0 of PMMA is more sensitive to the strain rate of the three materials and rapidly decreases with increasing strain rate, but the variation of the critical hydrostatic stress is similar to those of PC and PVC.

4. Discussion and conclusions

The mechanism of brittle fracture of notched ductile polymers in a thick plate form involves the formation of the plastic zone, followed by the nucleation of a disc-shaped craze at the tip of the plastic zone, and subsequent crack propagation around the disc. PC, PVC and quenched PMMA whose surfaces are carefully polished can be ductile in tension. However, as shown in the experimental results, if they contain a deep notch, they fracture in a brittle manner. In a state of plane strain the presence of the triaxial stress components raise the tensile stress level required to produce a given amount of shear yield stress at the notch root. Fracture of the specimen occurs when this tensile stress level ahead of the plastic zone is raised to an ideal fracture stress of the material. This is the basic explanation for notch brittleness as originally proposed by Orowan for mild steel [12]. An interpretative analysis of experimental results leads to the conclusion that Orowan's analysis is appropriate for notch brittleness of ductile glassy polymers. In glassy polymers, however, craze formation due to a hydrostatic stress component at the elastic–plastic boundary is responsible for brittle fracture.

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